Math 20550 - Calculus III
Exam 1 Review

12. Vectors and the Geometry of Space

12.1. Three-Dimensional Coordinate Systems.
- The distance formula between two points in $\mathbb{R}^3$
- The equation of a sphere

12.2. Vectors.
- How to make a vector between two points
- What it means for two vectors to be equal, parallel, or point in the same direction
- Vector arithmatic (addition, scalar multiplication, etc.)
- Magnitude of a vector
- Finding unit vectors in the direction of a given vector

12.3. The Dot Product.
- Definition of the dot product
- Properties of the dot product
- How to find the angle between two vectors
- How to check whether two vectors are perpendicular
- Projection of one vector onto another

12.4. The Cross Product.
- Definition of the cross product
- The fact that the resultant vector of a cross product is orthogonal to its factors
- Alternative formula for the angle between two vectors (involving the cross product)
- How to find the area of the parallelogram spanned by two vectors
- How to find the area of the triangle with vertices P, Q, R
- Properties of the cross product
- The triple scalar product
- How to find the volume of the parallelpiped spanned by three vectors

12.5. Lines and Planes.
- The vector equation for a line
- The parametric equations for a line
- The symmetric equations for a line
- How to find the equation of a line given
    - (1) a point on the line and a direction vector of that line
    - (2) two points on the line
- How to find the line segment between two points
- How to determine whether two lines are parallel, intersect, or skew
- The vector equation for a plane
- The scalar equation for a plane
- How to find the equation of a plane given
    - (1) a point on the plane and a vector normal to the plane
    - (2) two vectors and a point in the plane
    - (3) three points in the plane

(4) a line and a point in the plane
- Finding the point a line intersects a plane at (assuming they do intersect)
- How to determine whether two planes are parallel
- How to find the line two planes intersect along
- How to find the angle between two intersecting planes
- How to find the distance between a point and a plane
- The fact that the distance between a point P and a plane is the length of the vector which connects P and the point on the plane closest to P. Also that this vector is parallel to the normal vector of the plane.
- How to find the distance between two parallel planes

13. Vector Functions

13.1. Vector Functions and Space Curves.
- What a vector function is
- How to check if two curves intersect given their parametrizations
- How to take a limit of a vector valued function
- How to parametrize the intersection of two surfaces

13.2. Derivatives and Integrals of Vector Functions.
- How to find the derivative of a vector function
- How to find the tangent vector to a curve at a point on the curve
- How to find the tangent line to a curve at a point on the curve
- Properties of the derivative of vector functions
- How to find the unit tangent vector $\mathbf{T}$ to a curve
- How to find definite and indefinite integrals of vector functions
- Fundamental theorem of calculus for integrals of vector functions

13.3. Arc Length and Curvature.
- How to find the arc length of a curve
- How to find the arc length reparametrization (reparametrization with respect to arc length) of a curve
- How to find the curvature κ of a curve
- How to find the curvature of a plane curve $y = f(x)$
- How to find the unit normal $\mathbf{N}$ vector to a curve
- How to find the binormal vector $\mathbf{B}$ to a curve
- How to find the normal plane of a curve at a point on the curve
- How to find the osculating plane of a curve at a point on the curve

13.4. Motion in Space.
- How to find the velocity and acceleration functions of a particle given its position function
- Know what the speed of a particle is
- How to find the position function of a particle given its acceleration function and initial velocity and initial position
- How to find the tangential and normal components of acceleration
  (1) using speed ($\nu = |\mathbf{v}|$) and curvature (κ)
  (2) using the alternative formulas given